Joint Power Control and User Association in Downlink Heterogeneous Networks

David Ding

Supervisor: Prof. Wei Yu

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Introduction

Motivation

- Exponential growth of mobile traffic data: > 30.6 exabytes by 2020 [1]
- 5G: Next generation network designed to meet such demand with 1000x network capacity [2]
- Heterogeneous Network (HetNet): a promising technology to deliver high network capacity
  - Dense deployment of base stations (BSs) of varying transmit powers to cell edges
  - Brings users closer to access points → increase user rates

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HetNet Topology

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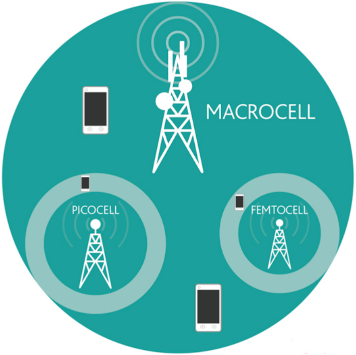

- BSs of different transmit powers are deployed densely
- Lower tier BSs refer to pico/femto BSs with much lower transmit power limits than that of macro BSs
- Main idea: low-tier BSs deployed near macro-cell edges to increase rates of cell-edge users by bringing those users closer to access nodes, without interfering with neighbouring cells

Challenges

- Macro-cell interference (intra-cell as well as inter-cell)
  - Future LTE/5G Networks: cell re-user factor of 1 to maximize the network capacity by maximizing bandwidth usage [3]
  - Macro BS transmit PSD limit >> pico/femto BS transmit PSD limit
- Under-utilisation of low-tier BSs
  - Shen and Yu [4] / Ye *et al.* [5] noted that higher SINR from macro BSs "attract" users
  - This in turn causes macro BSs to blast full power → interference causes pico BSs to yield low rates
- Network Consumption Power
  - Dense deployment of BSs → high total network consumption power
  - Wasteful if mobile traffic demand is low → how to dynamically adjust

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Related Works

On encouraging users to associate with low-tier BSs:

- Cell Range Expansion [5, 6, 7, 8, 9]: low-tier BSs receive a "biasing factor" to attract more users
  - Main drawback: still suffer from high interference of nearby macro BSs
- Heurestic association via techniques such as game theory [10, 11]: user-centric method where each users "bid" to be served by BSs. How much each users bid depends on the loading of BSs as well as rates other users can achieve while being served by this BS
- "Cost-Benefit" Analysis [4, 5]: loading of a BS treated as "cost" while instantaneous rates achieved by users served with that BS treated as "benefit". Users associate with a BS with highest "profit"
  - Ye *et al.* showed that proportional fairness is optimal scheduling scheme in single-input-single-output (SISO) downlink
  - Optimal algorithm if each BS in the end serves at least one user
  - However, if there are BSs that are idle (for example when number of users in the network is less or on the same order of magnitude as number of BSs), algorithm not optimal

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Related Works

Under proportional fairness scheduling: Both Shen and Yu and Ye *et al.* has objective function as $\sum_{i,j} x_{i,j} \log\left(\frac{W}{K_j}\log_2(1+\text{SINR}_{i,j})\right)$, where K_j is the number of users to be associated with BS j. Users are indexed with i and BSs are indexed with j. The binary association variable $x_{i,j} \in \{0,1\}$ indicates whether user i is served by BS j.

- Shen and Yu [4] proposed a distributed pricing-based association algorithm based on dual coordinate descent (DCD) applied on the dual of the original utility maximization problem
- Ye *et al.* [5] proposed a distributed user association algorithm based on primal-dual decomposition

In both cases, the "cost" of the BS is with regards to the variable K_j, of which the optimal value for each BS is determined in the distributed algorithms. However, due to the fact that K_j appears in the denominator of the utility expression, the case where there are idle BSs makes such algorithms sub-optimal.

Related Works

On mitigating interference for cell-edge users

- Soft Fractional Frequency Reuse (SFFR) [3, 12, 13, 14, 15]: macro cell divided into cell centre and cell edge regions, and allocate a portion of the spectrum for cell-edge users
  - Cell-edge spectrum has reuse factor greater than 1. Cell-centre users can access cell-edge spectrum of neighbouring cells with lower priority than cell-edge users from those cells.
  - How to best design the frequency reuse pattern for cell-edge users is unclear: trade-off between network capacity and cell coverage
  - Heuristic analysis of network performance [3] under cell-edge reuse factor of 3. Closed form analysis exist as well [12]
  - Heuristic greedy algorithm for user association and BS power control is state-of-art for both downlink [13] as well as uplink [14]
- Generalization of SFFR is to divide the entire transmission bandwidth into multiple sub-bands everywhere in the cell. To author's best knowledge, such generalization has not been properly analysed

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Related Works

On the optimization of user association with BS power control

- State-of-art is greedy BS selection [16, 17, 18, 19]: user association based on fixed BS power, then BSs are turned off/having power adjusted accordingly; the process is repeated until convergence. For example, Chiang *et al.* [16]'s model has BS transmit power divided into various power levels, and are adjusted accordingly depending on the result of previous user association. In Abbasi *et al.* [17], BSs are turned on one by one, and in each step, the BS providing the highest utility gain after user association is optimized is added to the active BS set. The algorithm terminates when no further BSs can be added to the active set
- These are effectively optimizing BS power control and user association in an iterative fashion.
  - Main drawback: Final user association dependent on initial SINR pattern. Common starting pattern is when all BSs are at full transmit power [4]. In this case, low-tier BSs are shut off prematurely and hence under-utilised

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Related Works

On the joint optimization of user association and BS power control

- In multiple-input-multiple-output networks involving beamforming, the appropriate formulation involves designing the optimal beamformer subject to maximization of weighted sum rates. In such considerations, BS activation algorithms have considered adding a power penalty in the form of L0 norm of beamformers tp the sum rate expression [19, 20, 21, 22]. The subsequent approximation of the L0 norm to other forms of differentiable norm motivates a weighted minimum mean square error (WMMSE) solution with the penalty term
- The idea is to find the best set of BSs to serve each user while limiting the total number of active BSs in the network
- However, to the best of author's knowledge, no equivalent tractable problem formulation has been derived for other forms of network utility, in particular, of log-utility

Central Question

How should users association and BS power control be jointly optimized to maximize network log-utility in a SISO, downlink HetNet, subject to potential network power consumption constraints?

Specifically, the thesis aims to use the optimization results to answer the following:

- How to formulate the user association problem under proportional fairness scheduling that takes into consideration of BSs potentially serving no users?
- Should a user, when provided the opportunity to associate with multiple BSs, still associate with a single BS that provides the highest rate to that user? **Short Answer: Yes**
- Can cell-edge users in a HetNet experience higher rates with the employment of FDMA in the downlink spectrum? **Short Answer: Yes**

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Central Question Cont'd

- Does the joint optimization of user association and BS power control provides significant rate increase compared to that from iterative optimization? **Short Answer: Yes**
- In a network where there is a total BS consumption power limit, can the HetNet provide the same network utility with tighter consumption power limits? In other words, does the interference reduction of some BSs turning off compensate for the reduced number of active BSs that must now serve all the users in the HetNet? **Short Answer: No**
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System Model

- SISO downlink HetNet of K users and B BS
- Assume flat-fading, time-invariant interference channel with channel side information (CSI) known at some central location
- FDMA-TDMA hybrid scheme possible: the total bandwidth, W, is divided into F sub-bands of equal bandwidth
  - Rationale: The ability to consider different values of F allows for comparison of performance between the multi-band ($F > 1$) and the single-band ($F = 1$) system
  - Helps to answer the question of whether HetNets provide higher rates to cell-edge users from dividing the entire downlink band into multiple sub-bands than without the sub-bands division
- Denote k, b, and f as indices over the set of K users, B BSs, and F sub-bands of transmission, respectively

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Channel Gains

Fading coefficients are denoted as $h_{k,b}^{f} \in \mathbb{C}$:

$$h_{k,b}^{f} = \sqrt{\xi_{k,b}}\phi_{k,b}^{f} \tag{1}$$

- $\xi_{k,b}$: large-scale fading dependent on the distance between user k and BS b, along with shadowing and antenna pattern (for sectorized cells)
- $\phi_{k,b}^{f}$: small-scale fading dependent on the arrival angles and scattering of the transmitted signals over each sub-band f from BS b and user k
  - Without loss of generality, assume circularly symmetric complex Gaussian with mean $\mathbf{0}$ and covariance matrix $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$
- $g_{k,b}^{f} = |h_{k,b}^{f}|^2$ denotes the channel gain–distinction between long-term and short-term

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Long term vs. Short term channel gain

- Small-scale fading coefficient generated over each signal transmission
- Time scale much too small for user association and BS power control optimization
- Hence, channel information used for optimization is $g_{k,b}^{f} = \xi_{k,b} \quad \forall f$, i.e., long-term channel gain
- Actual user achievable rates calculated via Monte-Carlo method where the gain is evaluated from various small-scale fading coefficients randomly generated, i.e. short-term channel gain

Network Utility Model

- Denote $r_{k,b}^f$ as the instantaneous rate that user k can achieve with BS b on sub-band f
- Denote $\mathbf{p} = [p_1^1, \cdots, p_1^F, \cdots, p_b^f, \cdots, p_B^F]^T$ as the stacked vector of $[p_b^f]$, where p_b^f denotes the downlink transmission PSD of BS b on frequency band f

$$r_{k,b}^f \triangleq \frac{W}{F} \log_2(1 + \gamma_{k,b}^f) \tag{2}$$

$$\gamma_{k,b}^f \triangleq \frac{g_{k,b}^f p_b^f}{\sigma^2 + \sum_{l \neq b} g_{k,l}^f p_l^f} \tag{3}$$

Long-term User Rates

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We model the long-term average user rates via the help of a "time-sharing" variable $\mathbf{x}$:

- Let a time slot over a sub-band of a BS be denoted as a "resource block" (RB). In practice, over one RB, only one user can be served (i.e. have the BS transmit signal to that user)
- However, the long-term average fraction of time slots user k is served by BS b on sub-band f can be denoted via $x_{k,b}^{f} \in [0, 1]$

Denote $\mathbf{x} = [x_{1,1}^{1}, \cdots, x_{1,1}^{F}, \cdots, x_{1_B}^{F}, \cdots, x_{k,b}^{f}, \cdots, x_{K,B}^{F}]^T$ as the stacked vector of the time-sharing variables $[x_{k,b}^{f}]$ (note that if $x_{k,b}^{f} = 0$, then it implies that user k is not served by BS b on slot f). The long-term rate user k achieves with BS b over all sub-bands is:

$$s_{k,b} = \sum_{f=1}^{F} x_{k,b}^{f} r_{k,b}^{f} \tag{4}$$

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Network Utility

It follows that the total user rate for user k attained over all frequency bands and by all BSs is thus:

$$R_k \triangleq \sum_{b=1}^{B} s_{k,b} \tag{5}$$

to which the network utility comes about as $\sum_{k=1}^{K} \log(R_k)$.

- This is log-utility, used to promote fairness to users with lower rates in the network
- This is particularly important in HetNet, as cell-edge users are encouraged to take advantage the deployment of low-tier access points to attain higher rates

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BS Power Model

We need a tractable model to examine the trade-off between network log utility and total BS consumption power

- Many examples in literature [17, 19, 23, 24] describes BS power consumption via two parts: transmit power and on-power:
- For example:

$$\begin{cases} \eta P_t + P_a, & \text{if } \; P_t > 0 \\ P_s, & \text{otherwise} \end{cases} \tag{6}$$

The model presented in this thesis is motivated from literature, and can be expressed directly from the optimization variable $\mathbf{p}$ using the L0-norm:

$$Q_b(\mathbf{p}_b) \triangleq \sum_{f=1}^{F} p_b^f + \psi_b ||\mathbf{p}_b||_0 \tag{7}$$

$$\Theta \triangleq \sum_{b=1}^{B} Q_b \tag{8}$$

Problem Formulation

The Main Problem

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{p}}{\text{maximize}} \quad & \sum_{k=1}^{K} \log(R_k) - \rho \sum_{b=1}^{B} Q_b(\mathbf{p}_b) && \text{(9a)} \\
\text{subject to} \quad & 0 \le p_b^f \le \bar{p}_b^f \quad \forall b,\ f && \text{(9b)} \\
& \sum_{k=1}^{K} x_{k,b}^f \le 1 \quad \forall b,\ f && \text{(9c)} \\
& x_{k,b}^f \ge 0 \quad \forall k,\ b,\ f && \text{(9d)}
\end{aligned}
$$

- Objective function (9a) describes the trade-off between network log-utility and total BS power consumption
- Parameter ρ controls the amount of power penalty applied in the trade-off
  - Note that $\rho = 0$ means the problem is a pure network utility maximization problem

The Main Problem

Optimization is over $\mathbf{x}$, the association variable, and $\mathbf{p}$, the BS PSD power control variable

- Constraint (9b) is the per BS, per sub-band PSD constraint
- Constraints (9c) and (9d) describe the resource constraint on each BS b for each sub-band f
  - the total fraction of time users spend on any RB cannot exceed one

Denote $\mathbf{x}^*$ as the optimization solution of problem (9) for user association. The rounding technique we use to obtain solution $\mathbf{y}^*$, that is, single-BS association solution, is:

$$y_{k,b}^{f*} = \begin{cases} x_{k,b}^{f*}, & \text{If } b = \arg\max_{l} \sum_f x_{k,l}^{f*} r_{k,l}^f \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

This allows us to compare single-BS and multiple-BS association results with power control.

- Similar "rounding" technique considered in Ye *et al.* [5] as "Fractional User Association"

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Problem Formulation Summary

The proposed problem is capable of answering all the thesis questions, as summarized:

- How to formulate the user association problem under proportional fairness scheduling that takes into consideration of BSs potentially serving no users?
  - Compare the rate performance of the problem formulation with time-sharing variable $\mathbf{x}$ with DCD algorithm
- Should a user, when provided the opportunity to associate with multiple BSs, still associate with a single BS that provides the highest rate to that user? **Short Answer: Yes**
  - This problem formulation readily presents a "single BS" solution from the multiple BS association solution via (10)
- Can cell-edge users in a HetNet experience higher rates with the employment of FDMA in the downlink spectrum? **Short Answer: Yes**
  - $F = 1$ for single-band, and $F > 1$ for multi-band

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- Should a user, when provided the opportunity to associate with multiple BSs, still associate with a single BS that provides the highest rate to that user? **Short Answer: Yes**
  - This problem formulation readily presents a "single BS" solution from the multiple BS association solution via (10)
- Can cell-edge users in a HetNet experience higher rates with the employment of FDMA in the downlink spectrum? **Short Answer: Yes**
  - $F = 1$ for single-band, and $F > 1$ for multi-band

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The proposed problem is capable of answering all the thesis questions, as summarized:

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Problem Formulation Summary Cont'd

- Does the joint optimization of user association and BS power control provides significant rate increase compared to that from iterative optimization? **Short Answer: Yes**
  - The proposed algorithm of joint optimization is readily compared with iterative update of $\mathbf{x}$ and $\mathbf{p}$ in terms of user rates
- In a network where there is a total BS consumption power limit, can the HetNet provide the same network utility with tighter consumption power limits? In other words, does the interference reduction of some BSs turning off compensate for the reduced number of active BSs that must now serve all the users in the HetNet? **Short Answer: No**
  - We can tweak the system parameter ρ to obtain the network utility given different BS power consumption limits and determine if tighter limits yield lower network utility rates (more details later)
- Do existing state-of-art greedy BS turn off algorithms provide comparable performances than direct optimization of user association and BS power control? **Short Answer: No**
  - The proposed algorithm is readily compared with existing greedy algorithms in terms of user rate performance

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Algorithm

Approximating L0-Norm

Recall the objective function:

$$\sum_{k=1}^{K} \log(R_k) - \rho \sum_{b=1}^{B} \left(\sum_{f=1}^{F} p_b^f + \psi_b w_b ||\mathbf{p}_b||_0 \right) \quad (11)$$

- The L0-norm term is integer and non-differentiable
- Motivated by Shi *et al.* [19], we can approximate the L0-norm by a *weighted mixed L1-L2 norm*, as follows: $w_b||\mathbf{p}_b||_2$
- This promotes *group sparsity* on $\mathbf{p}$ over groups of f, which is what we want
- The mixed L1-L2 norm has seen application in machine learning as the group least-absolute selection and shrinkage operator, and has been used with similiar effects on promoting sparse beamformers in MIMO systems [21]

Approximating L0-Norm

Applying the mixed norm technique, the following problem is an approximation to the main problem in (9).

$$\max_{\mathbf{x},\mathbf{p}} \quad \sum_{k=1}^{K} \log(R_k) - \rho \sum_{b=1}^{B} \left(\sum_{f=1}^{F} p_b^f + \psi_b w_b ||\mathbf{p}_b||_2 \right) \tag{12a}$$

$$\text{s.t.} \quad 0 \le p_b^f \le \bar{p}_b^f \quad \forall b,\ f \tag{12b}$$

$$\sum_{k=1}^{K} x_{k,b}^f \le 1 \quad \forall b,\ f \tag{12c}$$

$$x_{k,b}^f \ge 0 \quad \forall k,\ b,\ f \tag{12d}$$

The weights on L2-norm are updated as follows using techniques in [25]:

$$w_b = \frac{1}{||\mathbf{p}_b||_2 + \tau} \tag{13}$$

where $\tau > 0$ is a small regularization variable.

Joint Optimization over x and p

We then propose a hybrid "gradient-pseudo-Newton" projection method to update $\mathbf{x}$ and $\mathbf{p}$ jointly in order to maximize the objective function (12a) for any fixed ρ, now that the L0-norm has been properly approximated

- Main idea: take the gradient of (12a) with respect to $(\mathbf{x}, \mathbf{p})$ as well as the diagonal terms of (12a) with respect to $\mathbf{p}$
- Update $\mathbf{x}$ using the gradient as the search direction, since solving the gradient is of linear complexity with respect to K, B, and F, and project the updated $\mathbf{x}$ onto the constraints (12c) and (12d)
- Update $\mathbf{p}$ using a "pseudo-Newton" step to balance rate of convergence and computational complexity, as taking the gradient of objective with respect to $\mathbf{p}$ is quadratic in K for every b and f

Joint Optimization over $\mathbf{x}$ and $\mathbf{p}$

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The update direction with respect to $x^f_{k,b}$ is:

$$\Delta x^f_{k,b} = \frac{r^f_{k,b}}{\sum_{b=1}^{B} \sum_{f=1}^{F} x^f_{k,b} r^f_{k,b}} \tag{14}$$

The proposed algorithm updates each $x^f_{k,b}$ through:

$$x^f_{k,b}(t+1) = \mathbf{P}_{\Omega_{b,f}} \left(x^f_{k,b}(t) + \alpha_{\text{nt}} \Delta x^f_{k,b} \right) \tag{15}$$

Where $\mathbf{P}_{\Omega}(.)$ denotes the projection onto the set of probability simplex, Ω. Here, the probability simplex defined by $\Omega_{b,f}$ is simply:

$$\Omega_{b,f} = \begin{cases} \sum_k x^f_{k,b} \leq 1 \\ x^f_{k,b} \geq 0 \quad \forall k \end{cases} \tag{16}$$

- Such projection is well-studied in literature and works such as Wang and Carreira-Perpiñán [26] has provided an efficient algorithm for the projection
- The update step size, α_{nt}, can be determined via backtracking line search [27]

The update direction with respect to p_b^f is:

$$\Delta p_b^f = \frac{\partial f}{\partial p_b^f} \Big/ \left| \frac{\partial^2 f}{\partial (p_b^f)^2} \right| \tag{17}$$

With:

$$\begin{cases} \frac{\partial f}{\partial p_b^f} = \left\{ \sum_{k=1}^{K} \frac{\left(\frac{x_{k,b}^f}{(1+\gamma_{k,b}^f)}\right)\left(\frac{\gamma_{k,b}^f}{p_b^f}\right) - g_{k,b}^f \sum_{l \neq b} \left[\left(\frac{x_{k,l}^f}{(1+\gamma_{k,l}^f)}\right)\left(\frac{(\gamma_{k,l}^f)^2}{g_{k,l}^f p_l^f}\right)\right]}{R_k \log(2)} \right\} - \rho \left(1 + \psi_b w_b \frac{p_b^f}{||\mathbf{p}_b||_2}\right) \\ \frac{\partial^2 f}{\partial (p_b^f)^2} = \left\{ \sum_{k=1}^{K} \frac{A R_k - B}{R_k^2 \log(2)} \right\} - \rho \left(\psi_b w_b \frac{||\mathbf{p}_b||_2^2 - (p_b^f)^2}{||\mathbf{p}_b||_2^3} \right) \end{cases} \tag{18}$$

A and B are defined as follows:

$$\begin{cases} A = -\left(\frac{x_{k,b}^f}{(1+\gamma_{k,b}^f)} \frac{\gamma_{k,b}^f}{p_b^f}\right)^2 - (g_{k,b}^f)^2 \sum_{l \neq b} \left(\frac{x_{k,l}^f}{(1+\gamma_{k,l}^f)} \frac{(\gamma_{k,l}^f)^2}{g_{k,l}^f p_l^f}\right)^2 + 2(g_{k,b}^f)^2 \sum_{l \neq b} \left(\frac{(\gamma_{k,l}^f)^3}{(g_{k,l}^f p_l^f)^2}\right)\left(\frac{x_{k,l}^f}{(1+\gamma_{k,l}^f)}\right)^2 \\ B = \left(\frac{\partial f}{\partial p_b^f}\right)^2 \end{cases} \tag{19}$$

Where R_k represents the total long term rates for user k as defined in (5), and $\gamma_{k,b}^f$ denotes the receiver SINR between user k and BS b over sub-band f, as defined in (3).

The proposed algorithm updates all p_b^f through:

$$p_b^f(t+1) = \left[p_b^f(t) + \alpha_{\mathrm{nt}} \Delta p_b^f\right]_0^{\bar{p}_b^f} \tag{20}$$

The update step size, α_{nt}, can be determined via backtracking line search [27]

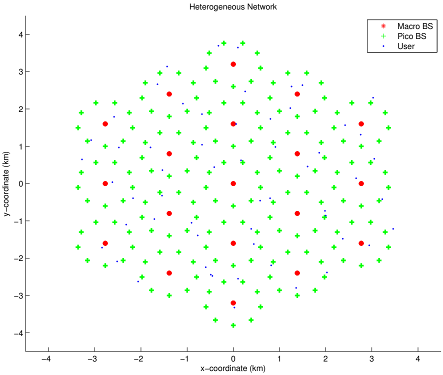
Algorithm 1 Joint Optimization of User Association and BS Power Control

INPUT: Channel Gains, Network Power Consumption Limit $\bar{P}$
OUTPUT: Optimal user association and scheduling $\mathbf{x}^*$ and optimal BS downlink transmit power $\mathbf{p}^*$

1: Pick large starting ρ
2: **repeat**
3: Initialize $\mathbf{x}$, $\mathbf{p}$ to feasible values
4: Initialize $\mathbf{w} = \mathbf{1}$
5: **repeat**
6: Solve problem (12) for fixed $\mathbf{w} = [w_b]$ and ρ using hybrid gradient-pN projection
7: Update w_b for all b according to (13)
8: **until** Maximum iterations reached, or convergence in (12a)
9: Compute the total network power consumption Θ using equation (8). If $\Theta < \bar{P}$, record down the network utility. Keep track of the best network utility achieved with $\Theta < \bar{P}$
10: Perform bi-section search update on ρ
11: **until** Maximum iterations reached, or network utility cannot be improved further, or $\Theta = \bar{P}$
12: If needed, apply (10) to obtain $\mathbf{y}^*$, the single-BS association solution from the multi-BS solution $\mathbf{x}^*$

Simulation Results

- Two-tiered HetNet containing 57 macro-cells with sectorization factor of 3
- One macro BS at the cell centre and three pico BSs in each cell near the cell-edge
- Path loss models are from standard 3GPP documents [28]
- Different BS on-powers are taken from [29]



Parameter	Value
Cell Layout	hexagonal
Tiers	2
Number of Macro BS	57
Number of Pico BS	171
Macro BS Density	1 per cell
Pico BS Density	3 per cell
Sectorization factor	3
Cell Re-Use Factor	1
Macro-Cell Radius	0.8 km
Pico-Cell Radius	0.2 km
Macro BS max transmit PSD	-27 dBm/Hz
Pico BS max transmit PSD	-47 dBm/Hz
Macro BS On-Power	1450 W
Pico BS On-Power	21.32 W
Channel Bandwidth	10 MHz
Noise PSD	-169 dBm/Hz
Noise figure	8 dB
Macro BS Pathloss	$128.1 + 37.6 \log_{10}(d)$ (dB)
Pico BS Pathloss	$140.7 + 36.7 \log_{10}(d)$ (dB)
Shadowing	8 dB
Fast Fading Coefficient	$\sim \mathcal{CN}(0,1)$

Simulation Overview

- Simulations showing multiple-BS vs single-BS association, multi-band vs. single-band, joint optimization compared to iterative optimization of $\mathbf{x}$ and $\mathbf{p}$, and comparison of user rates against DCD algorithm are investigated with exclusive focus on network utility maximization, and hence ρ is set to 0 throughout the simulations
- For the investigation of BS turn off patterns and the trade-off between network utility and BS power consumption, ρ is determined via the proposed algorithm 1 given the BS power consumption limit

Joint vs. Iterative Algorithm

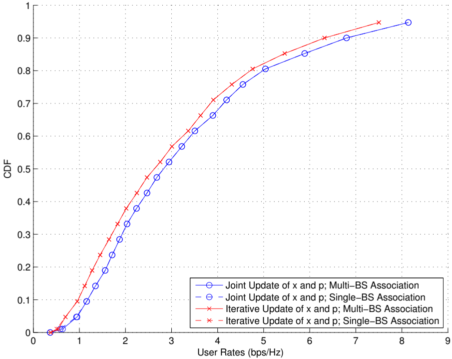


Figure: User CDF for $K = 57$, $F = 1$

- **Main Takeaway**: joint optimization improves rates over iterative optimization for all users

Single-BS vs Multi-BS

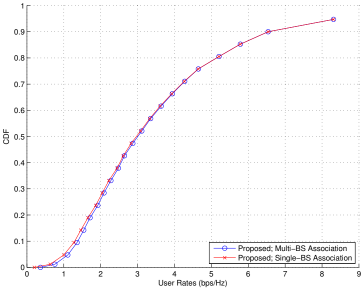


Figure: User CDF for $K = 57$, $F = 4$

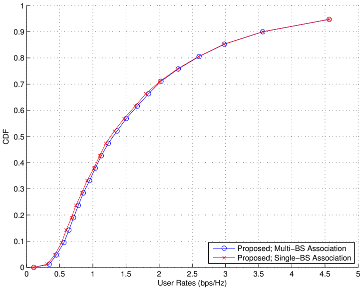
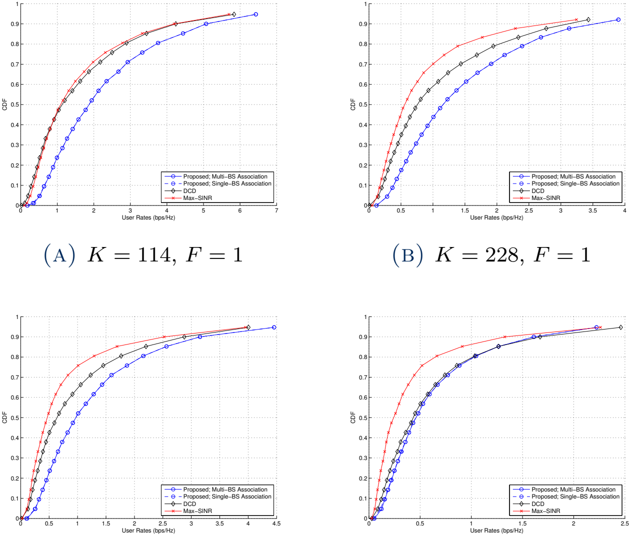


Figure: User CDF for $K = 228$, $F = 4$

- Assume no cooperation among multiple BSs serving a user
- Assume it is possible for a user to be served by multiple BSs on the same sub-band
- **Main Takeway**: users should be associated with a single-BS; algorithm provides confirmation of “rounding technique” being a tight upper bound for the integer problem of single-BS association in [5]

Figure: User CDF curves under single-band operations



(a) $K = 114,\ F = 1$

(b) $K = 228,\ F = 1$

(c) $K = 285,\ F = 1$

(d) $K = 570,\ F = 1$

Single-Band Operations

Main takeaway

Use of time-sharing variable is a proper way to formulate the network maximization problem for SISO downlink network that covers the cases when there are idle BSs

Multi-band vs. Single-band

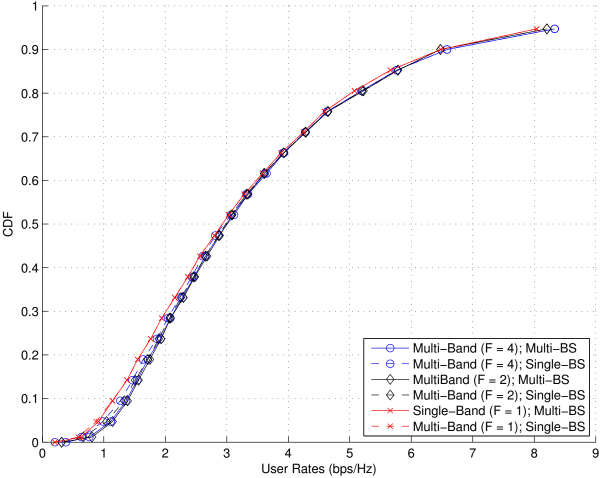


Figure: User CDF for $K = 57$

Multi-band vs. Single-band

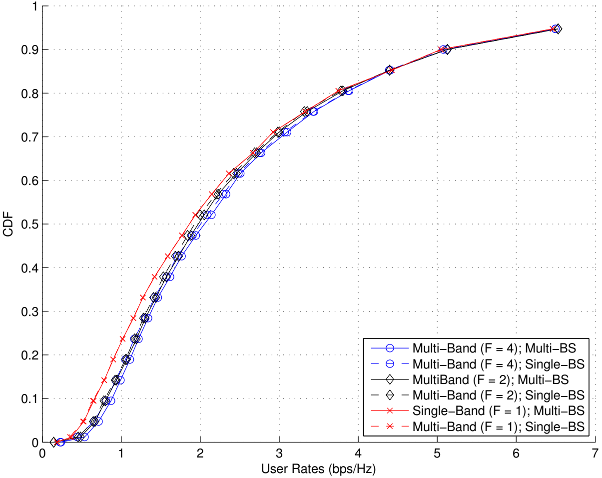


Figure: User CDF for $K = 114$

Multi-band vs. Single-band

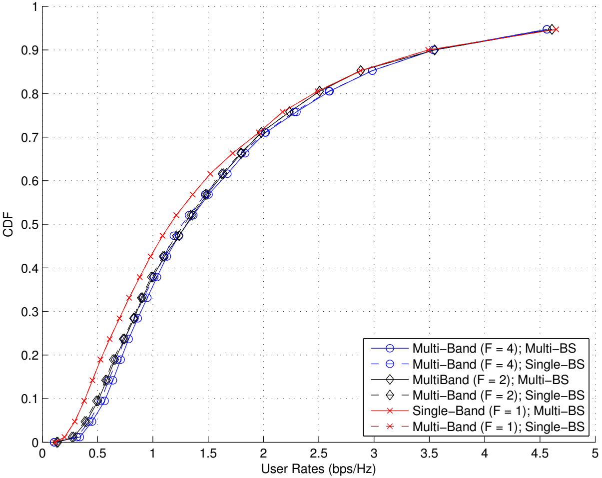


Figure: User CDF for $K = 228$

Multi-band vs. Single-band

Table: Percent Improvement of 10th Percentile Rates For Multi-band Over $F = 1$ Case

	F = 4	**F = 2**
K = 57	16.4	19.4
K = 114	22.1	32.5
K = 228	48.6	33.8

- **Main takeaway**: it is beneficial to perform FDMA in the network to benefit the cell-edge users
- In particular, benefit increases as number of users increases
- Number of sub-bands vs. cell-edge user rate has trade-off in complexity

BS Turn-Off

We compare the proposed algorithm with state-of-art greedy algorithms motivated in literature

- The greedy algorithm is one where initially users associate with the strongest BS, then, one by one, A BS with the least number of serving users is turned off and the affected users are re-associated to one of the remaining active BSs
- Two versions: on-off power (i.e. no power control) and with power control
- Denote $\bar{P}$ as the network power consumption limit
- Given $\bar{P}$, there is a corresponding ρ that is determined via bi-section search in the proposed algorithm 1

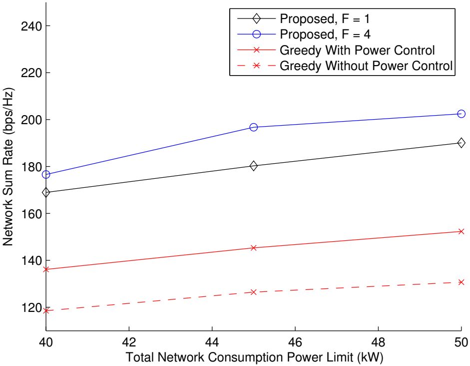


Figure: Network Sum Rate vs. $\bar{P}$ for $K = 57$

BS Turn-Off

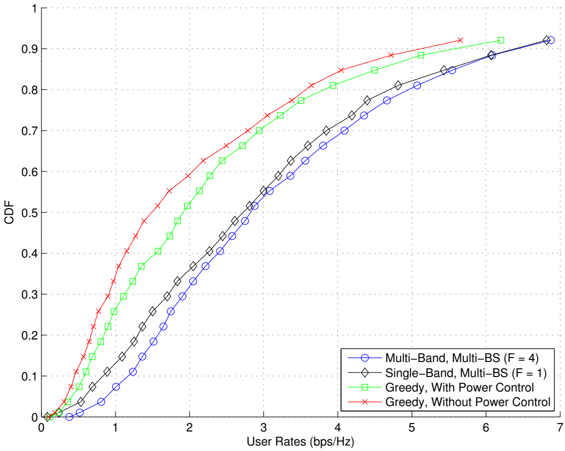


Figure: User CDF Curves For $\bar{P} = 50{,}000$ W, $K = 57$

BS Turn-off

Main Takeaways:

- Cell-edge users continue to see rate improvement for multi-band over single-band for various network power consumption limits
- As we decrease the network power consumption limit, we see a decrease in network utility. This indicates that the interference reduction from turning off BSs does not compensate properly for the increased loading of the remaining active BSs
- Turning off BSs prematurely has a negative impact on network utility, as the proposed algorithm's performance gap over the greedy algorithms illustrate

CONCLUSION

Main Results

- Joint optimization of user association and BS power control yields significantly higher network utility than greedy benchmarks or iterative optimization in a HetNet due to the effectiveness for joint optimization to promote network heterogeneity
- Users should be associated with a single BS in a network that provides the highest long term rates
- The formulation of the network utility maximization problem should be where the association and scheduling are represented via a "time-sharing" variable to take care of potential idle BSs in the optimization
- Cell-edge users benefit in rates in an FDMA system
- The interference reduction from turning off BSs does not compensate properly for the increased loading of the remaining active BSs

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Future Works

- Consideration of joint optimization of user association and beamforming in MIMO systems
- What if BSs can cooperate?

[1] "Cisco visual networking index: Global mobile data traffic forecast update, 2015–2020," Cisco, White Paper 1460442270491203, February 2016.

[2] Huawei, "5g: A technology vision," Tech. Rep., 2013. [Online]. Available: http://www.huawei.com/5gwhitepaper/

[3] Y. Yu, E. Dutkiewicz, X. Huang, M. Mueck, and G. Fang, "Performance analysis of soft frequency reuse for inter-cell interference coordination in lte networks," in *2010 International Symposium on Communications and Information Technologies (ISCIT)*, Oct 2010, pp. 504–509.

[4] K. Shen and W. Yu, "Distributed pricing-based user association for downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1100–1113, June 2014.

[5] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis, and J. G. Andrews, "User association for load balancing in heterogeneous cellular networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, pp. 2706–2716, June 2013.

[6] D. Lopez-Perez, X. Chu, and İ. Guvenc, “On the expanded region of picocells in heterogeneous networks,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 281–294, June 2012.

[7] R. Madan, J. Borran, A. Sampath, N. Bhushan, A. Khandekar, and T. Ji, “Cell association and interference coordination in heterogeneous lte-a cellular networks,” *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1479–1489, December 2010.

[8] D.-H. Sung, J. S. Baras, and C. Zhu, “Coordinated scheduling and power control for downlink cross-tier interference mitigation in heterogeneous cellular networks,” in *2013 IEEE Global Communications Conference (GLOBECOM)*, Dec 2013, pp. 3809–3813.

[9] K. Okino, T. Nakayama, C. Yamazaki, H. Sato, and Y. Kusano, “Pico cell range expansion with interference mitigation toward lte-advanced heterogeneous networks,” in *2011 IEEE International Conference on Communications Workshops (ICC)*, June 2011, pp. 1–5.

[10] K. Han, D. Liu, Y. Chen, and K. K. Chai, “Energy-efficient user association in hetnets: An evolutionary game approach,” in *2014 IEEE*

Fourth International Conference on Big Data and Cloud Computing (BdCloud), Dec 2014, pp. 648–653.

[11] M. Chiang and J. Bell, "Balancing supply and demand of bandwidth in wireless cellular networks: utility maximization over powers and rates," in *2004 INFOCOM. Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies*, vol. 4, March 2004, pp. 2800–2811 vol.4.

[12] T. D. Novlan and J. G. Andrews, "Analytical evaluation of uplink fractional frequency reuse," *IEEE Transactions on Communications*, vol. 61, no. 5, pp. 2098–2108, May 2013.

[13] M. Qian, W. Hardjawana, Y. Li, B. Vucetic, X. Yang, and J. Shi, "Adaptive soft frequency reuse scheme for wireless cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 1, pp. 118–131, Jan 2015.

[14] X. Mao, A. Maaref, and K. H. Teo, "Adaptive soft frequency reuse for inter-cell interference coordination in sc-fdma based 3gpp lte uplinks," in *2008 IEEE Global Telecommunications Conference*, Nov 2008, pp. 1–6.

[15] S. Kumar, S. Kalyani, and K. Giridhar, “Optimal design parameters for coverage probability in fractional frequency reuse and soft frequency reuse,” *IET Communications*, vol. 9, no. 10, pp. 1324–1331, 2015.

[16] P.-H. Chiang, P.-H. Huang, S.-S. Sun, W. Liao, and W.-T. Chen, “Joint power control and user association for traffic offloading in heterogeneous networks,” in *2014 IEEE Global Communications Conference (GLOBECOM)*, Dec 2014, pp. 4424–4429.

[17] A. Abbasi and M. Ghaderi, “Energy cost reduction in cellular networks through dynamic base station activation,” in *2014 Eleventh Annual IEEE International Conference on Sensing, Communication, and Networking (SECON)*, June 2014, pp. 363–371.

[18] K. Son, H. Kim, Y. Yi, and B. Krishnamachari, “Base station operation and user association mechanisms for energy-delay tradeoffs in green cellular networks,” *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1525–1536, September 2011.

[19] Y. Shi, J. Zhang, and K. Letaief, “Group sparse beamforming for green cloud-ran,” *IEEE Transactions on Wireless Communications*, vol. 13, no. 5, pp. 2809–2823, May 2014.

[20] M. Hong, R. Sun, H. Baligh, and Z.-Q. Luo, "Joint base station clustering and beamformer design for partial coordinated transmission in heterogeneous networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 226–240, February 2013.

[21] W. C. Liao, M. Hong, Y. F. Liu, and Z. Q. Luo, "Base station activation and linear transceiver design for optimal resource management in heterogeneous networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 15, pp. 3939–3952, Aug 2014.

[22] W. C. Liao, M. Hong, and Z. Q. Luo, "Base station activation and linear transceiver design for utility maximization in heterogeneous networks," in *2013 IEEE International Conference on Acoustics, Speech and Signal Processing*, May 2013, pp. 4419–4423.

[23] S. Ping, A. Aijaz, O. Holland, and A. H. Aghvami, "Green cellular access network operation through dynamic spectrum and traffic load management," in *2013 IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Sept 2013, pp. 2791–2796.

[24] K. Dufková, M. Bjelica, B. Moon, L. Kencl, and J. Y. L. Boudéc, "Energy savings for cellular network with evaluation of impact on data traffic performance," in *2010 European Wireless Conference (EW)*, April 2010, pp. 916–923.

[25] E. J. Candes, M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted l1 minimization," *Journal of Fourier Analysis and Applications*, vol. 14, no. 5-6, pp. 877–905, 2008. [Online]. Available: http://dx.doi.org/10.1007/s00041-008-9045-x

[26] W. Wang and M. Á. Carreira-Perpiñán, "Projection onto the probability simplex: An efficient algorithm with a simple proof, and an application," *CoRR*, vol. abs/1309.1541, 2013. [Online]. Available: http://arxiv.org/abs/1309.1541

[27] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[28] 3GPP, "Further advancements for e-utra physical layer aspects," Tech. Rep. TS36.814 v9.0.0.

[29] O. Arnold, F. Richter, G. Fettweis, and O. Blume, "Power consumption modeling of different base station types in heterogeneous cellular networks," in *2010 Future Network Mobile Summit*, June 2010, pp. 1–8.