Throughput Analysis of Cognitive Radio-Based Small Cells in LTE Downlink Networks

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***Abstract*—As the advancement of wireless communication technologies continue to proliferate in the early part of the 21st century, significant attention is devoted towards designs of the next generations of cellular networks. An integral part of the new network is small cells, which are created to improve cellular capacity. At the same time, radio spectrum is severely limited due to the static bandwidth and the growth of wireless services. A novel concept of using cognitive radio (CR) technology to establish small cell access points (APs) has been introduced to combat both issues. This paper aims to investigate important aspects of CR-based small cell networks designed to improve cellular capacity without requiring extra bandwidth. This paper examines spectrum detection methods by the CRs implemented onto the small cell APs to maximize small cell throughput as secondary users in the cellular network under the constraint of primary user performance. Results of this paper can be extrapolated to optimize the operation of CR-based small cells.**

I. INTRODUCTION

In order to meet the demands for growing usage of mobile phones, small cells have being introduced to improve the capacity of cellular network. Small cells improve cellular capacity by covering smaller physical regions than macrocells but are capable of handling roughly the same number of users as macrocells. According to a study conducted by Qualcomm, four small cells placed inside a macrocell can increase the capacity of the region covered by the macrocell by 2.8 fold [1].

However, small cell nodes potentially require extra bandwidth for transmission, and it is desired that transmissions of data from small cell nodes do not require extra bandwidth. In recent years, the radio spectrum has become valuable resources, and by 2020, wireless traffic will increase by a thousand fold [2]. It is highly desirable if transmissions of small cell nodes can occur in the same radio spectrum as that for macrocell nodes without reducing the quality of service (QoS) of the macrocell users.

Cognitive radio is a piece of technology that has been brought up to mitigate the issues of spectrum crowding. The CR is a wireless device that is capable of detecting available wireless channels for data transmission, and tune its carrier and modulation scheme to transmit on those channels. This allows small cell nodes to transmit on the same spectrum as macrocell users by exploiting the spatial-temporal holes on the spectrum. However, this can only be done if small cell network APs cause minimal to no interference against the macrocell users. The reason is because the cellular bands are initially licensed to macrocell for transmitting data, which makes the macrocell users as primary users (PUs). Secondary Users (SUs) refer to small cell AP users who are using this licensed band as unlicensed users, and from [3], [4], they must do so in a manner that maintains the required QoS for the PUs.

This paper investigates the performance of CR-based small cell APs (CR-APs) in terms of throughput under the constraint set by the PUs. The investigation is conducted under the context of LTE downlink network and assumes the CR-APs do not know any information about the PU *apriori*. The metric for measuring QoS for the PUs is the PU throughput in the downlink under the presence of SUs. In literature, various spectrum detection rules concerning method of combining joint detection results [5] and local fusion performance [6] have been investigated. In addition, many spectrum allocation methods aimed to maximize throughput under OFDM systems have been devised through various algorithms such as the Graphing Colouring Model [7], Particle Swarm Operation [8] and waterfill algorithms for Frequency Hopping OFDM [9]. This paper combines the two topics through an analysis of the performance of outdoor small cells under the LTE network. In this paper, the investigation of various CR spectrum detection methodologies is conducted, and the resulting optimal rule in terms of spectrum detection accuracy is used to analyze the maximum allowable throughout of the small cell downlink network under the minimum allowed decrease in PU throughput. According to [5], the joint detection combination for optimal detection is that of equal gain combining (EGC) when no channel state information (CSI) is provided at the receiver. Similarly, energy detection is the optimal detection method for CRs with no CSI available. Since this paper investigates CR-APs with no knowledge of CSI, the CRs analyzed are operating using energy detection and individual detection results are combined via EGC.

The paper is organized as follows: Section II describes the system model and mathematical theory of the spectrum detection and throughput analysis. Section III analyzes the simulation results of the model described in Section II. Finally, Section IV concludes the paper.

II. SYSTEM MODEL

This section outlines the system models and theory applied to the analysis of the performance of CR-APs. First, the system setup for the detection of spectrum is described, followed by the probability theories involved in CR energy detection. Then, the system setup for the cellular environment containing both the small cells as well as their positions within the macrocells is presented for throughput analysis.

A. Spectrum Detection

The CR topology considered here is cooperative sensing via a fusion centre (FC). Such considerations were made in light of the realistic assumption that individual CRs could be affect by the hidden node problem. Cooperative sensing in this topology would take advantage of the spatial diversity of the CRs and mitigate the effects of the hidden node problem. In order for the detection accuracy to improve for the overall system compared to individual detection, multiple CR-APs are connected to an FC. In this paper, the number of connected CRs considered is 10.

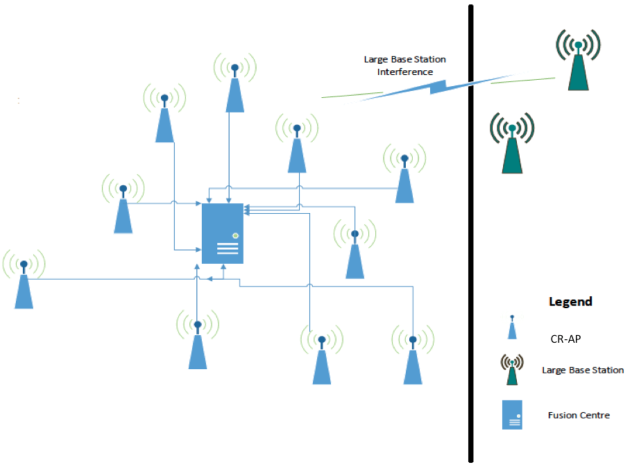

Fig. 1. CR Spectrum Detection Topology.

Figure 1 illustrates the CR detection topology setup involving the FC. The FC communicates with the CR-APs in reading individual detection results and broadcasting final decision results for spectrum detection. The CR-APs are placed at random positions. Each CR-AP is placed inside a small cell, and the positions of the CR-APs are modeled by a random variable uniformly distributed between 470m and 500m that represents the distance between the access point and the large base station (BS).

The large BS is considered a PU in the cellular spectrum used by CR-APs, and the CR-APs are the SUs. The large BS transmission could potentially interfere with the SUs, if the CRs incorrectly detect the occupied PRBs as free. On the other hand, the noise at the front-end of the CR receiver could cause the CRs to falsely determine a PRB occupied by PU when it is in fact free of PU activity. The level of threshold determines probabilities of have a false-alarm detection or a missed detection.

In order to consider detection performance, the following system parameters are used:

TABLE I
SYSTEM PARAMETERS FOR SPECTRUM DETECTION

Parameter	Value
One-Sided Noise Power Spectrum Density (N0)	-174 dBm/Hz
Number of PRBs in an OFDM system	64
PRB Bandwidth	180 KHz
Noise Figure	8dBm
Total Noise Power	-50.8dBm
Maximum BS Cell Size	500 m
BS Cell Frequency Re-Use Cluster Size	3
Sectors/BS Cell	3
Maximum Outdoor Small Cell Size	30 m
Total BS Transmit Power	46 dBm
Total CR-AP Transmit Power	20 dBm
Number of individual samples	3

The following path loss model and parameter is used for considering the large scale fading of the PU signal being detected by the CR-AP:

$$PL(d_{out}) = 38.76 + 37.6log_{10}(d_{out}) + L_s[dB] \quad (1)$$

with the following parameters:

TABLE II
BS TO CR-AP PATH LOSS MODEL PARAMETERS

Parameter	Value
d_{out}: distance between the outdoor CR-AP the large BS	$\mathcal{U}(470, 500)$ m
L_s: shadowing loss	$\mathcal{N}(0, 6)$ dB

Mathematical Theory of Spectrum Detection: Consider N CR-APs connected to an FC, where decisions on whether to transmit on an OFDM PRB are made. From co-operative sensing, the FC employs the k-out-of-N rule, where if at least k of the N CR-APs detect that the spectrum is in use by the PU, then the spectrum is in use and is not available for transmission of SU data. The following values of k define the rules:

Rule	k
Or-Rule	1
And-Rule	N
Majority Rule	$\lceil \frac{N}{2} \rceil$

Definition 1: H_0 = the null hypothesis on the OFDM PRB that the PU is not transmitting on this PRB.

Definition 2: H_1 = the alternative hypothesis on the OFDM PRB that the PU is transmitting on this PRB.

Using energy sensing, denote the i^{th} sample taken by a particular CR-AP to be a power reading P_i. Let each CR-AP take m samples and ϵ is the average energy reading.

Definition 3:

$$\epsilon = \frac{1}{m}\sum_{i=1}^{m} P_i \quad (2)$$

Definition 4: p_{flocal} = the probability of false alarm at each CR-AP.

It follows from 4 that:

$$p_{flocal} = Pr[\epsilon > \theta | H_0] \quad (3)$$

Where θ is some pre-determined threshold.

p_{flocal} is the same for all N CR-APs, since under H_0, the only source of power detected is AWGN, which is assumed to have the same power under the same spectrum bandwidth.

Definition 5: p_{fa} = the probability of false alarm at the FC, assuming k-out-of-N rule.

It follows that:

$$p_{fa} = Pr[X > k] = 1 - F_X(k-1; N, p_{flocal}) \quad (4)$$

Where X is an RV representing the total number of CR-APs in the network of N CR-APs connected to FC showing false alarm. Since all CR-APs exhibit the same probability of false alarm, X is a binomial RV, with parameters N and p_{flocal}. $F_X(x; N, p)$ is the corresponding CDF of X.

Since the noise considered is AWGN, we denote w_i as the i^{th} power sample taken by the CR-AP under H_0. Thus, $w_i \sim \mathbb{C}\mathcal{N}(0, \sigma_w^2)$. Using this fact and equation (2), we have:

$$\frac{2}{\sigma_w^2} \sum_{i=1}^{m} |w_i|^2 \sim \ \chi^2(2m) \quad (5)$$

Combining (5) with (3) give rise to the following expression for p_{flocal} in terms of the CDF of an RV of $2m$ degrees of freedom[1]:

$$p_{flocal} = Q(m, \frac{m\theta}{\sigma_w^2}) \quad (6)$$

Using (6) and (4), we arrive at an equation showing the relationship between the threshold θ and p_{flocal}:

$$p_{fa}(\theta) = 1 - F_X(k-1; N, Q(m, \frac{m\theta}{\sigma_w^2})) \quad (7)$$

Where:

$$\theta = \frac{\sigma_w^2 \times Q^{-1}(m, p_{flocal})}{m} \quad (8)$$

Now, under H_1, denote x_i to be the PU signal power for the i^{th} CR detection sample, denoted as P_i. Here it is assumed that the PU signal is independent of noise. Assume the PU signal is Gaussian, i.e., $x_i \sim \ \mathbb{C}\mathcal{N}(0, P)$, where P is the average power of the PU signal, we arrive at:

$$P_i = |x_i + w_i|^2 \sim \ exp(P + \sigma_w^2) \quad (9)$$

Definition 6: p_{dlocal} = the local probability of detection at each CR-AP.

From 6, it follows that:

$$p_{dlocal} = Pr[\epsilon > \theta | H_1] \quad (10)$$

Using (9), (10) and similar analysis with probability of false alarm, we arrive at the following expression for p_{dlocal} in terms of the CDF of a chi-squared RV with $2m$ degrees of freedom:

$$p_{dlocal} = Q(m, \frac{m\theta}{P + \sigma_w^2}) \quad (11)$$

Since the CR-APs are positioned at different locations with respect to the large BS, their corresponding average PU signal power readings are therefore different, and hence each CR-AP has a different p_{dlocal}. We denote the p_{dlocal} for the l^{th} CR-AP to be p_{dl} and the corresponding average PU signal power detection to be P_l. We thus have:

$$p_{dl} = Q(m, \frac{m\theta}{P_l + \sigma_w^2}) = Q(m, \frac{m\theta}{\sigma_w^2(1+\gamma_l)}) \quad (12)$$

Where γ_l is the average SNR detected by the l^{th} CR-AP.

With the k-out-of-N rule, let p_d be the probability of detection at the FC:

$$p_d = \sum_{j=k}^{N} \binom{N}{j} \prod_{l \in R} p_{dl} \prod_{l \in T} (1 - p_{dl}) \quad (13)$$

Where R is the set of CR-APs that correctly detected spectrum occupied by the PU, while T is the set of CR-APs that did not. Note we have $|R| + |T| = N$.

B. Throughput Analysis

Figure 2 presents the system setup for the outdoor CR-APs. Here a realistic scenario is considered, whereby the CR small cells are situated near macrocell boundaries. The purpose of the small cells is to serve users near macrocell boundaries where, due to path loss and multipath fading, the signal reception tends to be the poorest. At the macrocell boundaries, interference from multiple BSs must be considered. Hence, the paper factors not only the large BS from the macrocell containing the small cell, but also neighbouring large BSs. For this paper, we are only considering single small cell user case.

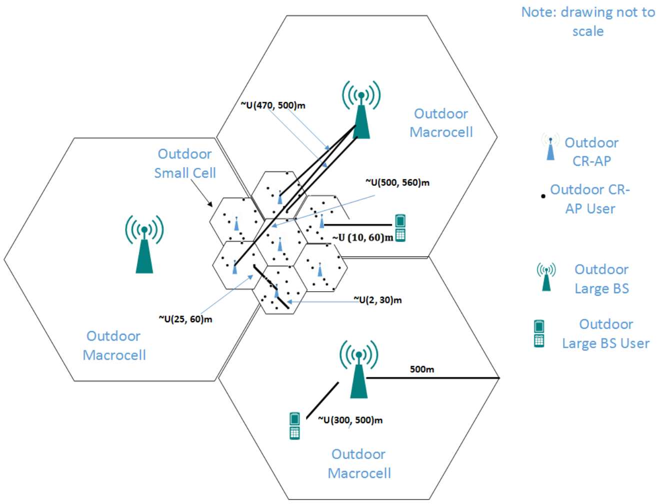

Fig. 2. Outdoor CR-AP Network Setup.

Several path loss models are considered here. The general path loss equation is given in equation (1). The parameters d_{out} and L_s is described in different scenarios as follows.

First, we consider the downlink interference between a CR-AP and a BS user, i.e. the PU:

Parameter	Value
d_{out}: distance between the PU and the outdoor CR-AP	$\mathcal{U}(10, 60)$ m (Note: the paper is considering PU near the outdoor CR-AP for interference analysis)
L_s: shadowing loss	$\mathcal{N}(0, 6)$ dB

Then, we have the path loss between BS and the PU:

Parameter	Value
d_{out}: distance between the PU and the BS	$\mathcal{U}(300, 500)$ m (Note: the paper is considering PU near the outdoor CR-AP for interference analysis)
L_s: shadowing loss	$\mathcal{N}(0, 6)$ dB

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[1] Complementary CDF of a chi-squared RV with $2k$ degrees of freedom is $Q(k, x) \equiv \frac{\Gamma(k,x)}{\Gamma(k)}$

Interference between BS and SUs is considered in two cases: the SUs situated in own macrocell and SUs situated in neighbouring macrocells.

Parameter	Value
d_{out}: distance between the SU and the BS	$\mathcal{U}$(470, 500) m for SU within BS macrocell; $\mathcal{U}$(500, 560) m for SU within neighbouring BS macrocells
L_s: shadowing loss	$\mathcal{N}$(0, 6) dB

Distance between PU and BS:

Parameter	Value
d_{out}: distance between the PU and the BS	$\mathcal{U}$(300, 500) m (Note: the paper is considering PU near the outdoor CR-AP for interference analysis)
L_s: shadowing loss	$\mathcal{N}$(0, 6) dB

Finally, we shall consider the throughput of the CR-AP with consideration of the interference from BS.

Parameter	Value
d_{out}: distance between the CR-AP and own SU	$\mathcal{U}$(2, 30) m for own SU; $\mathcal{U}$(25, 60) m for other SUs not within own small cell
L_s: shadowing loss	$\mathcal{N}$(0, 6) dB

Mathematical Theory of Throughput: Once the FC made decision of the state of each OFDM PRB, it will assign each CR-AP to transmit on the detected free PRBs with a certain probability, taken in this paper to be $(macrocell\ reuse\ cluster\ size)^{-1}$. Denote $C_{l,i}$ to be the throughput of the l_{th} CR-AP transmitting to its single user on the i^{th} PRB, we have:

$$\begin{aligned} C_{l,i} &= log_2(1+SINR_{l,i}) \\ &= log_2\left(1+\frac{|H_i|^2 Pr_{l,i}}{W+I_{PU,l,i}+\sum_{j\neq l} I_{SU,l,j}}\right) \end{aligned} \tag{14}$$

Where I is the interference caused either by the PU or other SU transmitters. H_i is the small scale fading value of the i^{th} PRB, and $Pr_{i,l}$ is the received power of the l_{th} SU from the l_{th} CR-AP on the i^{th} PRB.

The optimization problem is outlined in (15):

$$\max_{p_{fa}} \sum_{i\in R}\sum_{l\in N_i} C_{l,i} \tag{15}$$

$$\text{subject to: } \begin{cases} C_{l,i} = log_2(1+SINR_{l,i}) \\ \sum_{i\in S} log_2\left(1+\frac{|H_i|^2 Pr_i}{W+\sum_{l_i} I_{SU,l,i}}\right) \geq R_{\text{acceptable}} \end{cases} \tag{16}$$

The objective function in problem (15) denotes the maximization of the SU throughput, which is largely decreasing with p_{fa}, because as p_{fa} increases, fewer free PRBs are detected to be free by the FC and therefore more CR-APs are forced to transmit on a single PRB, causing stronger SU interference, I_{SU}, and therefore decreasing the throughput as seen in (14). Although increase in p_{fa} also decreases I_{PU}, the effect is not as significant as that for I_{SU} because the PU signal experiences much deeper fading than SU signals. On the other hand, as p_{fa} decreases, the macro-BS throughput decreases due to increased CR-AP activity, and we have a trade-off between SU rate and PU rate. Since the spectrum puts PU rate as priority, the SU usage of the channels over this spectrum cannot decrease the PU rate by the pre-determined acceptable QoS rate, denoted as $R_{\text{acceptable}}$.

III. RESULTS

Figure 3 shows the corresponding receiver operating characteristics (ROC) curve of the CR-AP under different rules, under the constraint that the macrocell has radius of 500m as described in Section II.

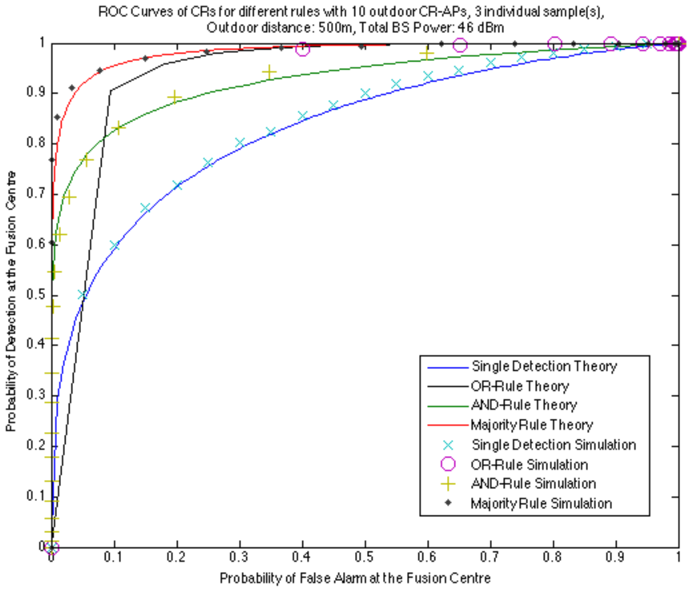

Fig. 3. ROC for CR-AP, both Theory and Simulation

From the ROC curve, it is shown that all joint detection rules perform better in terms of accuracy of spectrum detection compared to single detection. Furthermore, the Majority Rule has the best performance among all the joint detection rules considered. Hence, for throughput analysis, the Majority Rule is used for the FC to decide which OFDM PRBs are free for the CR-APs to transmit.

In (12), each p_{dl} can be expressed as a function of the average SNR detected. From the simulation, it is found that the ROC of all CR-AP can be fitted with a curve as follows:

$$p_{dlocal} = Q(m, \frac{m\theta}{\sigma_w^2(1+\bar{\gamma})}) \tag{17}$$

The value of $\bar{\gamma}$ in this case is found to be 1. This means regardless of the position of the CR-APs with respect to the large BS, there exists an average SNR value that can fully characterize the performance of spectrum detection of the CR-AP network.

Using (17), the interference between the SU and PU is analyzed for two spectrum states: 90% and 75% PU spectrum occupancy. Figure 4 shows the throughput of the PU as function of p_{fa}.

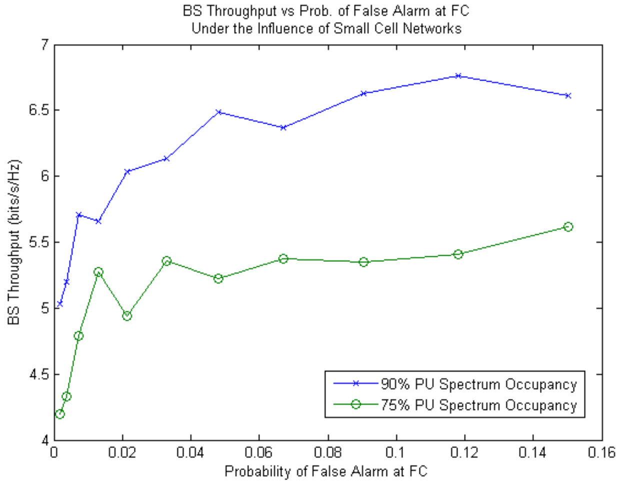

Fig. 4. Throughput of large BS vs p_{fa} Simulation Results

And the corresponding CR-AP throughputs are shown in figure 5.

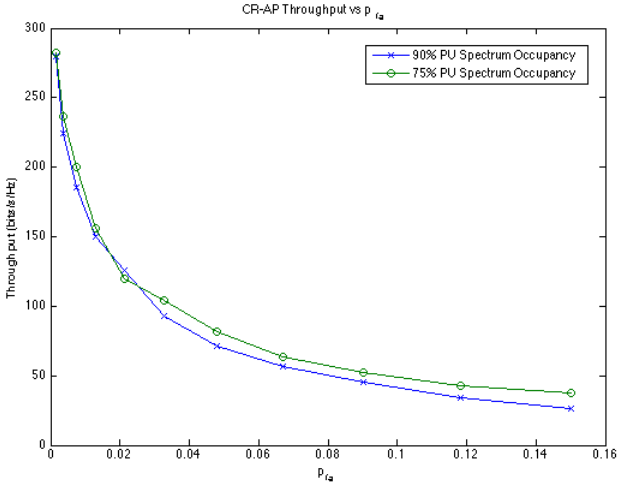

Fig. 5. Throughput of CR-AP vs p_{fa} Simulation Results

From figure 3, using the majority rule, the probability of detection for the CR-AP is > 0.95, thus showing almost perfect detection when $p_{fa} > 0.15$, and hence we can assume that the BS throughput has reached maximum value at $p_{fa} = 0.15$ on average. In figure 4, assume the PU can tolerate at most a 5% drop in throughput performance, then the minimum acceptable PU throughput is around 6.2bits/s/Hz for 90% occupancy, achieved with $p_{fa} = 0.04$ and 5.2 bits/s/Hz for 75% occupancy, achieved with $p_{fa} = 0.016$. At these occupancy rates, referring to figure 5, with $p_{fa} = 0.04$, the maximum throughput that can be achieved for the CR-AP network as a whole is around 95 bits/s/Hz for 90% occupancy. With $p_{fa} = 0.016$, the maximum throughput that can be achieved for the CR-AP network as a whole is around 120 bits/s/Hz for 75% occupancy. Note that the simulation agreed with the theory that p_{fa} postively correlates with BS throughput and negatively correlates with CR-AP, and thus the trade-off scenario indeed yields an optimal p_{fa} value based on the constraints set by the PU. Furthermore, while there is a huge performance gap between the spectrum occupancy rates for the BS throughput, the gap is less noticeable for the CR-AP. This is caused by the drop in I_{PU} due to the increase in spectrum detection accuracy that compensated for the increase in I_{SU} as p_{fa} increased.

IV. CONCLUSION

This paper determined the theoretical bounds achieved in downlink transmission rates for the small cell network opportunistically accessing the PU spectrum, based on optimal detection rules from joint spectrum detection. It provides an insight on the expected capacity of the future network infrastructure as well as the thresholds for the CRs to operate to potentially achieve the throughput. The investigation not only focuses on the maximum rates achieved by the SUs, but also considering the impact on the PUs to reflect the limitations of the SUs in the spectrum and thereby obtaining realistic results complying with the regulations of unlicensed users.

In the future, a possible extension to this paper would be to investigate ways in which the throughput bounds can be approached as closely as possible. The use of antenna arrays can also be investigated to increase the gains of the antennae in the networks. Terminal-aided spectrum detection can also be examined as possible methods to improve spectrum detection even further through the exploitation of greater spatial diversity. Here, Doppler effects must be considered, which extends to this paper that only considered static nodes in the downlink system. One can also examine the throughputs of the small cell networks from an uplink perspective.

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